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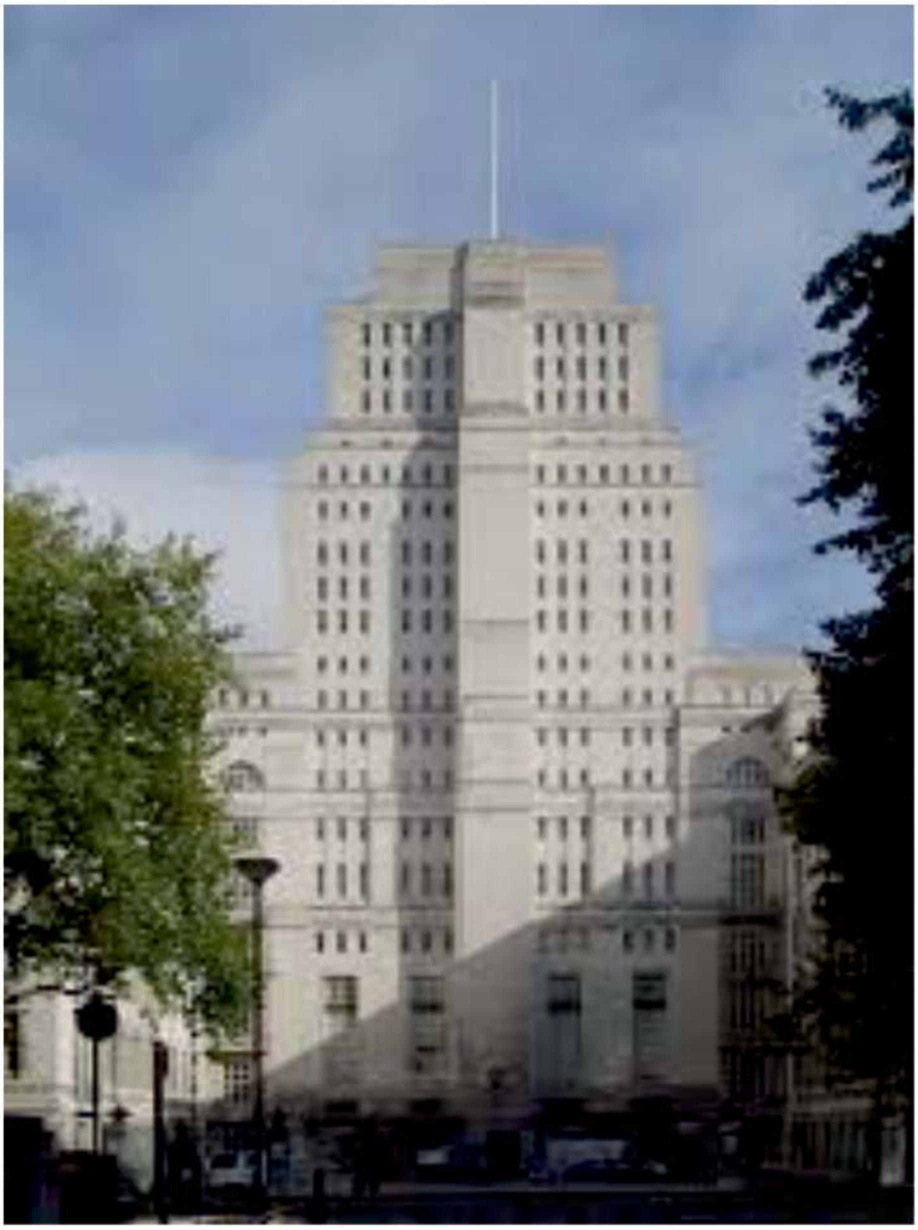

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Economics

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Management and optimization of financial flows

Abstract: The algorithm of the solution of a problem of dynamic management the period of 2011-2016 is considered in this work. For decision the method of the principle of a maximum of Pontryagin is used. The equation of a state registers concerning Gross Domestic Product. The General government total expenditure function is used As management.

Keywords: Management, profit, GDP, the Pontryagin maximum principle
JEL classification codes: C01; C53; E17; F21

Introduction

Management of financial flows presupposes the equation of state, management functions, as well as the optimality criterion. One method of solving the management problem is the method of the Pontryagin maximum principle [1,2,3,4].

In this article we propose an algorithm for solving the management problem, which is carried out in several stages:

1. Determine the optimal functional dependence on the known (discrete) initial data;
2. Finding the corresponding differential equation;
3. Definition of the management function and the optimality criterion;
4. Determination of management function for the task.

The advantages of this approach are:

1. Ease of use;
2. Availability of obtaining raw data (eg [5]);
3. Use in different areas;
4. Real numerical results.

The process of finding the optimal functional relationship between the variables can be found in [6,7].

Problem statement

As a function of the equation of state will use the Gross Domestic Product (GDP). Depending on the processed raw data we obtain the corresponding equation of state and initial conditions.

As a management function $U(t)$ the value of general government total expenditure (% GDP) is used [5]:

$$U(t) = \alpha_2(t)Y(t), \quad \alpha_2(t) > 0 \qquad (1)$$

Optimality condition is maximization the discounted difference between the values of investment (% GDP) -$\alpha_1(t)$, general government revenue (% GDP) - $\alpha_3(t)$ and general government total expenditure (% GDP) - $\alpha_2(t)$ for a certain period and has the form:

$$\int_{t_0}^{T} \exp(-\delta t)(\alpha_1(t) + \alpha_3(t) - \alpha_2(t))Y(t)dt \to \max \qquad (2)$$

As an example, consider the problem of optimal management with the initial data for United Kingdom [5] on the interval $[t_0; T]$:

Table 1: Initial data for calculation

t	Год	GDP (Current Prices, US$ Billion), *Y(t)*	Investment (% of GDP), $\alpha_1(t)$	General government total expenditure (% of GDP), $\alpha_2(t)$	General government revenue (% of GDP), $\alpha_3(t)$
1	2011	2471,88	15,584	45,861	37,301
2	2012	2602,49	16,411	44,48	37,542
3	2013	2743,35	17,358	42,793	37,776
4	2014	2890,99	18,18	41,26	37,843
5	2015	3050,52	18,913	40,072	37,779
6	2016	3220,42	19,601	38,704	37,384

Processing the data in Table 1, we find the optimal function in the form $Y = ab^t$. Applying the method of the least squares, we obtain the unknown parameters: $a = 2342{,}0198$; $b = 1{,}053276$. This optimal function corresponds to the following boundary value problem:

$$\frac{dY(t)}{dt} = \ln bY(t)\,;\; Y(t_0) = ab \qquad (3)$$

Equation (3) is the equation of studied system state, with the initial condition.

Find the optimal functional dependence for $\alpha_2(t)$- general government total expenditure (% of GDP) on the interval $t \in [t_0; T]: \alpha_2(t) = 47{,}2492 - 1{,}444057t$

(4)

Thus, we obtain a mathematical formulation of the overall cost management problem for the state:

The equation of state of the system:

$$\frac{dY(t)}{dt} = \ln bY(t) + U(t) \qquad (5)$$

Initial conditions:

$$Y(t_0) = ab \qquad (6)$$

Management function has following form:

$$U(t) = \alpha_2(t)Y(t) \qquad (7)$$

where $\alpha_2(t)$ has the form (8).

Optimality condition is given in the form:

$$\int_{t_0}^{T} \exp(-\delta t)(\alpha_1(t) + \alpha_3(t) - \alpha_2(t))Y(t)dt \to \max \qquad (8)$$

Thus, it is necessary to maximize the integral equation (8) when the conditions (4) - (7) are fulfilled.

To solve the problem we apply the Pontryagin maximum method [3].

We write the Hamiltonian function:

$$H(t) = \Psi(t)\{\alpha_2(t) + \ln b\}Y(t) + \exp(-\delta t)\{\alpha_1(t) + \alpha_3(t) - \alpha_2(t)\}Y(t) \quad (9)$$

where $\Psi(t)$ - auxiliary function that satisfy equation

$$\frac{d\Psi(t)}{dt} = -\Psi(t)\{\alpha_2(t) + \ln b\} - \exp(-\delta t)\{\alpha_1(t) + \alpha_3(t) - \alpha_2(t)\} \quad (10)$$

For the auxiliary function transversality is carried out

$\Psi(T) = 0$

By analyzing the Hamiltonian (10) we obtain an optimal investment strategy:

$$\alpha_2(t) = \begin{cases} \max \alpha_2(t) = \alpha_2(t_0)\,,\, t_0 \le t \le t_1^* \\ \min \alpha_2(t) = \alpha_2(T) \,,\, t_1^* < t \le T \end{cases} \quad (11)$$

where t_1^* - time of switching company's investments, which is found from condition

$$\Psi(t_1^*) - \exp(-\delta t_1^*) = 0 \quad (12)$$

Auxiliary variable $\Psi(t)$ in the interval $[t_0; t_1^*]$ in the management $\alpha_2(t) = \alpha_2(t_0)$ is determined by solving the boundary problem:

$$\begin{cases} \dfrac{d\Psi(t)}{dt} = -\Psi(t)\{\alpha_2(t_0) + \ln b\} - \exp(-\delta t)(\alpha_1(t) + \alpha_3(t) - \alpha_2(t_0)) \\ \Psi(t_1^*) = \exp(-\delta t_1^*) \end{cases} \quad (13)$$

Solution (13) assuming

$$\begin{aligned} \alpha_1(t) &= \max \alpha_1(t) = \alpha_1 = const \\ \alpha_3(t) &= \max \alpha_3(t) = \alpha_3 = const \end{aligned} \quad (14)$$

Has the form:

$$\Psi(t) = \exp\{-\delta t_1^* + (\alpha_2(t_0) + \ln b)(t_1^* - t)\}(1 - A) + A\exp(-\delta t) \quad (15)$$

where $A = \dfrac{\alpha_1 + \alpha_3 - \alpha_2(t_0)}{\delta - \alpha_2(t_0) - \ln b}$

On interval $[t_1^*; T]$ with the management $\alpha_2(t) = \alpha_2(T)$ function $\Psi(t)$ is determined by solving the problem:

$$\begin{cases} \dfrac{d\Psi(t)}{dt} = -\Psi(t)\{\alpha_2(T) + \ln b\} - \exp(-\delta t)(\alpha_1(t) + \alpha_3(t) - \alpha_2(T)) \\ \Psi(T) = 0 \end{cases} \quad (16)$$

The solution (16) with constraints (14) has the form:

$$\Psi(t) = \frac{\alpha_1 + \alpha_3 - \alpha_2(T)}{\delta - B}\{\exp(-\delta t) - \exp[-T(\delta + B) + Bt]\} \quad (17)$$

where $B = \alpha_2(T) + \ln b$

Time of switching company's investments is obtained from condition (12)

$$t_1^* = T - \frac{1}{\delta - \alpha_2(T) - \ln b}\ln\left|\frac{\alpha_1 + \alpha_3 - \alpha_2(T)}{\alpha_1 + \alpha_3 - \delta + \ln b}\right| \quad (18)$$

Numerical examples

As a numerical experiment, we use the following inputs:

$\delta = 0{,}05$; $\alpha_1 = 0{,}19601$; $\alpha_3 = 0{,}37843$; $t_0 = 1$; $T = 6$; $Y(1) = 2466{,}79$

The optimal management function (7) that achieves the maximization of (8) is found. The switch point is equal to $t_1^* = 3{,}11$. Analyzing obtained results, we propose the following distribution of General government total expenditure (% of GDP): on the period 2011-2013,11 plan 45.826%, and during 2013,11-2016- 38.704%. In this manner the cumulative discounted value, which is consists of the sum of Investment and General government revenue minus the General government total expenditure, will be maximum over the period 2011-2016.

Conclusion

An algorithm for solving the problem of dynamic financial flows management of the state is described. Solution of the optimal management problem is carried out using the method of Pontryagin maximum principle. It is shown that the proposed algorithm works and has practical application. As an example, the problem of optimal components management of GDP for the United Kingdom during 2011-2016 is considered.

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